

# Model-independent view on the low-mass proton-antiproton enhancement

B.Kerbikov, A.Stavinsky, V.Fedotov

State Research Center

Institute of Theoretical and Experimental Physics,  
Moscow, Russia

## Abstract

We present a simple interpretation of the recently observed near-threshold proton-antiproton enhancement. It is described by a set of low-energy parameters deduced from the analysis of  $N\bar{N}$  experiments at LEAR. We predict a related effect in photoproduction reaction under study by CLAS collaboration.

Low-mass baryon-antibaryon enhancement has recently been observed in the decays  $B^+ \rightarrow K^+ p\bar{p}$  [1],  $\bar{B}^0 \rightarrow D^0 p\bar{p}$  [2] and  $J/\psi \rightarrow \gamma p\bar{p}$  [3]. On the theoretical side this discovery has been discussed by several authors [4, 5, 6, 7]. In [4] the near-threshold effect has been considered either as a gluonic state or as a result of the quark fragmentation process. In [5] it has been attributed to the Breit colorspin interaction while in [6] it has been regarded as caused by peripheral one-pion-exchange potential. The quantum numbers corresponding to the observed enhancement are discussed in [7].

The purpose of the present work is to show that the recently observed near-threshold  $p\bar{p}$  enhancement may be understood invoking the knowledge on  $N\bar{N}$  interaction gained at the Low Energy Antiproton Ring (LEAR) at CERN. The LEAR results may be schematically subdivided into three groups:

- (i) meson spectroscopy,
- (ii) baryonium searches,
- (iii)  $N\bar{N}$  interaction: scattering, annihilation, protonium spectroscopy.

A comprehensive review of the investigations performed at LEAR along these lines may be found e.g. in [8]. At first sight it might seem tempting to connect the newly observed structure with the elusive baryonium [9]-[11]. The point of view adopted in this paper is different. We remind that baryonium searches at LEAR ended without any clear evidence for its existence [8]. On the other hand the studies of  $N\bar{N}$  elastic scattering, annihilation and charge-exchange reaction along with the protonium studies yielded reliable information on low-energy  $N\bar{N}$  amplitude [8]. It has been shown [12]-[15] that the whole set of the experimental data on low-energy  $N\bar{N}$  interaction is possible to describe in terms of the effective range approximation. In this approach the lack of the dynamical picture is traded for the possibility to reconcile within a unique scheme different pieces of information on  $N\bar{N}$  interaction.

We shall see that the effective range analysis [12]-[15] based on the old LEAR data enables to explain Belle [1, 2] and BES [3] results. In particular the observation by BES of a strong effect in the decay  $J/\psi \rightarrow \gamma p\bar{p}$  and the absence of a similar structure in  $J/\psi \rightarrow \pi^0 p\bar{p}$  perfectly fits into the solution for the low-energy parameters obtained 15 years ago [12]-[15]. We shall return to this point below.

The problem of whether any conclusions on baryonium can be inferred from the low-energy parameters is beyond the scope of this work. We only mention that the values of the scattering lengths presented below are not anomalously large as it should be for the case of a level close to threshold. One must also keep in mind that strong annihilation drastically changes the standard relation between the low-energy parameters and the positions of the poles of the amplitude [16].

Our approach is based on the low-energy analysis of final-state  $p\bar{p}$  interaction. Therefore the near-threshold enhancement observed in different reactions [1, 2] is described by the same equations presented below. Another consequence of the proposed scheme is the prediction of the similar phenomena in other reactions of the same type, in particular in photoproduction  $\gamma p \rightarrow pp\bar{p}$ . This process can be investigated by CLAS collaboration at Jefferson Lab. Unlike  $B$ - and  $J/\psi$  decays the photoproduction reaction has not been discussed from the theoretical side. Therefore we shall choose it as a starting point in order to introduce the effective range formalism. Then we shall return to Belle and BES data.

The double differential cross section for the reaction  $\gamma p \rightarrow pp\bar{p}$  is given

by the well-known Chew-Low expression

$$\frac{d^2\sigma}{ds_2 dt_1} = \frac{1}{2^{12}\pi^4} \frac{\lambda^{1/2}(s_2, m^2, m^2)}{m^2 k^2 s_2} \int d\Omega_{23}^* |T(s_2, t_1, \Omega_{23}^*)|^2, \quad (1)$$

where  $m$  is the nucleon mass,  $k$  is the energy of  $\gamma$ ,

$$s_2 = (p_2 + p_3)^2, \quad t_1 = (k - p_1)^2, \quad (2)$$

with  $p_2$  and  $p_3$  being the 4-momenta of  $p$  and  $\bar{p}$  forming the low-mass pair,  $p_1$  being the 4-momentum of the remaining proton,  $\lambda(x, y, z) = (x - y - z)^2 - 4yz$  is the standard kinematical function. The angle  $\Omega_{23}^* = (\cos\theta, \varphi)$  is defined in the CM system of the particles (2,3). The variable  $s_2 = m_{23}^2$  is the square of the invariant mass of the (2,3) pair. It follows from (3) that the angular distribution of particles 2 and 3 ( the low-mass  $p\bar{p}$  pair) is isotropic in their CM system provided  $T(s_2, t_1, \Omega_{23}^*)$  is independent of  $\Omega_{23}^*$  which is the case for  $^1S_0$  and  $^3P_0$  states. Equation (1) yields the following invariant mass distribution

$$\frac{d\sigma}{dm_{23}} = \frac{1}{2^{10}\pi^4} \frac{|\mathbf{p}_1|}{km^2} (m_{23}^2 - 4m^2)^{1/2} \int d(\cos\beta) \int d\Omega_{23}^* |T|^2, \quad (3)$$

where  $\beta$  is the angle between  $\mathbf{p}_1$  and the direction of the incident  $\gamma$  ( we remind that the index 1 is attributed to the proton which remains outside the correlated  $p\bar{p}$  pair).

In the spirit of the Migdal-Watson FSI theory we single out from the matrix element  $T$  a factor responsible for the low energy  $p\bar{p}$  interaction. Within Migdal-Watson approach one has

$$|T|^2 = |T^{(0)}|^2 / |f(-q)|^2 = D(q) |T^{(0)}|^2, \quad (4)$$

where  $q$  is the  $p\bar{p}$  CM momenta,  $q^2 = \frac{1}{4}(m_{23}^2 - 4m^2)$ ,  $f(-q)$  is the Jost function corresponding to the  $p\bar{p}$  interaction at low energy,  $D(q) = |f(-q)|^{-2}$  is called the enhancement factor. As it was already mentioned the dynamics of  $N\bar{N}$  interaction is much more complicated than that of  $NN$  [8], [9], [11], [16]. Annihilation dominates at short and possibly intermediate distances,  $\omega$ - and other odd  $G$ -parity exchanges lead to a strong attraction to be added to the one-pion exchange in the outer region [17]. Therefore the approximation of  $f(-q)$  by a Born term from one-pion exchange [6] misses the essential features of  $N\bar{N}$  dynamics. On the other hand effective range solution for

the low-energy  $N\bar{N}$  amplitude describes the whole set of data with a fair accuracy [12]-[15].

Ignoring for the moment complications due to annihilation, Coulomb interaction and spin-isospin structure we may write the following expression for Jost function in the scattering length approximation

$$f(-q) \simeq A(q)(1 - iqa), \quad (5)$$

where  $a$  is the scattering length and  $A(q)$  is a well defined smooth function. A similar expression for  $p\bar{p}$  system is not as simple for three reasons:

- (i)  $p\bar{p}$  system is a combination of the two isospin states with  $I = 0, 1$ ; isospin invariance is violated by the mass difference of proton and neutron;
- (ii) a powerful annihilation results in the complexity of the scattering lengths;
- (iii) the Coulomb attraction acts in the  $p\bar{p}$  system.

Experimental data on  $N\bar{N}$  spin observables exist only at the incident  $\bar{p}$  momenta  $P_L > 500$  MeV/c [18, 8], i.e. beyond the energy region where the effective range approximation is applicable. From the level shifts in  $p\bar{p}$  atom [19] it is possible to extract only some information on the imaginary parts of the triplet and singlet scattering lengths [20]. Therefore only spin-averaged effective range parameters were extracted from the experimental data [12]-[15].

The enhancement factor in which all three points (i) -(iii) inherent for the  $p\bar{p}$  system are accounted for has the following form [12]-[15]

$$D(q) = \frac{c^2(q)}{|1 - is(\tilde{q} + i\Delta + l) - (\tilde{q} + i\Delta)lr|^2}, \quad (6)$$

where

$$c^2(q) = \frac{2\pi}{qa_B} \left[ 1 - \exp\left(-\frac{2\pi}{qa_B}\right) \right]^{-1} \quad (7)$$

is the Sakharov Coulomb enhancement factor [21] with  $a_B = 2/\alpha m = 57.6$  fm being the Bohr radius of the  $p\bar{p}$  atom,

$$s = \frac{1}{2}(a_0 + a_1), \quad r = a_0 a_1, \quad (8)$$

where  $a_I, I = 0, 1$  are  $N\bar{N}$   $S$ -wave scattering lengths with isospin  $I$ ,

$$\tilde{q} = c^2(q)q + \frac{2i}{a_B}h(qa_B), \quad h(z) = \ln z + \text{Re}\psi\left(1 + \frac{i}{z}\right), \quad (9)$$

with  $\psi(z) = d/dz \ln \Gamma(z)$ . The quantity  $l$  is the momentum in the  $n\bar{n}$  channel,

$$l^2 = q^2 - m\delta, \quad \delta = 2(m_n - m), \quad (10)$$

(recall that  $m$  is the proton mass).

The point  $q = (m\delta)^{1/2} \simeq 49$  MeV/c corresponds to the  $n\bar{n}$  threshold; below this point  $n\bar{n}$  momentum  $l$  becomes imaginary,  $l = i(m\delta - q^2)^{1/2}$ . Parameter  $\Delta$  is the Schwinger correction to the scattering length [22],  $\Delta \simeq -0.08$  fm [12]-[15].

The enhancement factor (6) does not include the effective range term and the contribution from nonzero orbital momenta. According to [12]-[15] the effective range term is of minor importance up to  $q \simeq 150$  MeV/c i.e. to  $Q = m_{23} - 2m \simeq 20$  MeV. We note in passing that for a multichannel system the effective range may be even negative or complex [23, 24]. The  $P$ -wave comes into play a little earlier especially as far as the total cross section is concerned. Inclusion of  $P$ -wave brings about two problems. First, the  $P$ -wave scattering length (it has a dimension of fm<sup>3</sup>) is very sensitive to the fitting procedure (see [12]-[15]) and the present set of the experimental data do not warrant a stable solution for the  $l = 1$  amplitude. The second problem is the following. With  $P$ -wave included equations become rather cumbersome and the number of parameters increases substantially. This may cause unnecessary doubts in the reliability of the proposed approach. We plan to consider effective range terms and  $l > 0$  amplitudes in the next publication. This would allow to analyze the  $p\bar{p}$  correlation function in a wider energy range.

The most remarkable result of all fits [12]-[15] for low energy  $N\bar{N}$  parameters is a clear dominance of the  $S$ -wave with  $I = 0$  over that with  $I = 1$ . The corresponding scattering lengths read

$$a_0 = (-1.2 + i0.9) \text{ fm}, \quad a_1 = (-0.1 + i0.4) \text{ fm}, \quad (11)$$

$$a_0 = (-1.1 + i0.4) \text{ fm}, \quad a_1 = (0.3 + i0.8) \text{ fm}, \quad (12)$$

where (11) and (12) are respectively the results of [13] and [14]. The sign convention in [12]-[15] is  $k \cot \delta = +1/a$ . We see that the absolute value of  $|Rea_0|$  is much larger than that of  $|Rea_1|$ . This is completely in line with the observation by BES of a strong low-mass effect in the decay process  $J/\psi \rightarrow \gamma p\bar{p}$  and the absence of a similar structure in  $J/\psi \rightarrow \pi^0 p\bar{p}$  [3].

The two solutions given by (11) and (12) are somewhat different. This is due to several reasons described in [12]-[15]. The main source of ambiguity

is the lack of the experimental data at very low energies. The effective range analysis which resulted in the parameter sets (11)-(12) was performed at the end of eighties. In view of the LEAR shut down the experimental situation did not drastically change since that time. The only substantial new piece of information concerns the total and annihilation  $\bar{p}p$  and  $\bar{n}p$  cross sections down to  $p_{Lab} = 35$  MeV/c [25, 26]. However in the analysis of these data [27, 28] use was made of the effective range solution [29] performed in 1988 which was later updated in 1991 [14] partly by the original authors. The parameters set (12) is just this updated version. The effective range solution [29] fairly well fits the low energy cross sections [25]-[28] which is an indication that the same is true for our parameter set (12). We urge our colleagues who are in possession of the experimental data to persuade the analysis with the parameter sets [13, 14]. An independent phenomenological analysis of the experimental results [25] was performed in [30]. The authors extracted the imaginary part of the Coulomb-distorted  $p\bar{p}$  scattering length. This imaginary part corresponds to the averaging over spin and isospin. The  $n\bar{n}$  threshold which lies just within the region under investigation was not taken into account. This fact should be kept in mind when comparing our parameter sets with the value presented in [30]. The result of [30] is  $Ima_{cs} = 0.69$  fm while from (11) we get  $\frac{1}{2}Im(a_0 + a_1) = 0.65$  fm (the index in  $a_{cs}$  means that this quantity still contains the Coulomb correction). Again we conclude that the “new” data [25] do not contradict the “old” solution (11).

The negative sign of  $Rea_0$  may be interpreted as an indication that the potential in this channel is either repulsive or on the contrary attractive and strong enough to produce a bound state somewhere below the threshold. However such a simple interpretation is not obvious in strong annihilation regime. This problem is beyond the scope of the present work.

The above arguments based on the analysis of the reaction  $\gamma p \rightarrow p\bar{p}$  led us to the following conclusion. Within the framework of the FSI theory and within the region of the applicability of the scattering length approximation the  $p\bar{p}$  effective mass distribution is governed by the function

$$F(Q) = (m_{p\bar{p}}^2 - 4m^2)^{1/2} D(Q) \simeq 2(mQ)^{1/2} D(Q), \quad (13)$$

where

$$Q = m_{p\bar{p}} - 2m = 2m \left\{ \left( 1 + \frac{q^2}{m^2} \right)^{1/2} - 1 \right\}, \quad (14)$$

and  $D(Q)$  is defined by (6). The phase-space factor  $Q^{1/2}$  is trivial and plays the role of the universal background. The function  $D(Q)$  reflects the the main features of the  $p\bar{p}$  dynamics. Due to Coulomb attraction it diverges at threshold since  $C^2(q) \rightarrow 2\pi/a_B q$  as  $q \rightarrow 0$ . Thus at  $Q = 0$  the distribution  $2(mQ)^{1/2}D(Q)$  starts from a finite value  $4\pi/a_B$ . At  $q = (m\delta)^{1/2}$  the factor  $D(Q)$  has a cusp due to the opening of the  $n\bar{n}$  channel.

The above conclusions also hold true for the reactions  $B^+ \rightarrow K^+ p\bar{p}$ [1],  $\bar{B}^0 \rightarrow D^0 p\bar{p}$ [2] and  $J/\psi \rightarrow \gamma p\bar{p}$ [3].

An remark is due at this point. The BES data on  $J/\psi \rightarrow \gamma p\bar{p}$  were analyzed making use of the Breit-Wigner formula [3]. We wish to remind that in the vicinity of the threshold the Breit-Wigner function can be reexpressed in terms of the effective range parameters [31].

Our main results are presented in Figs. 1-2. In Fig. 1 we display the enhancement factor  $D(Q)$  corresponding to the parameters set (11). In this figure we also plot the experimental points obtained by BES collaboration and taken from Ref.[3]. The normalization of the experimental points is chosen in such a way that in the region of  $Q \simeq 0.02$  GeV they are in rough agreement with the theoretical curves. The comparison therefore may serve only for the orientation purposes and in order to give an impetus for the future detailed analysis of Belle and CLAS data as soon as these data are available.

The curve  $D(Q)$  corresponding to the parameter set (12) is not shown since it goes very close to the presented in Fig.1. In Fig. 2 we illustrate the roles played by the Coulomb attraction and annihilation. The solid curve in Fig.2 is the same as in Fig.1, the dashed one corresponds to the Coulomb interaction switched off, while the dotted curve – to switching off the annihilation.

Our main conclusions may be summarized as follows. The near-threshold proton-antiproton enhancement has been observed in reactions driven by different mechanisms. Therefore the effect may be due to  $p\bar{p}$  FSI. In order to verify this assumption we have described the FSI in terms of the low-energy  $N\bar{N}$  parameters taking into account the isospin structure of the amplitude and the Coulomb interaction. The factorization of the FSI factor from the total amplitude has been explicitly performed for the reaction  $\gamma p \rightarrow p p\bar{p}$ . It turns out that the data of BES collaboration [3] are fairly well described by the effective range solution obtained long ago from the analysis of the LEAR data. Further conclusions would become possible as soon as new experiment data from BES, Belle and CLAS are available. Finally we remark that similar

kind of analysis may be applied to  $\Lambda\bar{\Lambda}$  final states. The low energy parameters for this system may be found e.g. in [32].

The authors are indebted to L.N.Bogdanova, A.Kudryavtsev, K.Mikhailov, P.Pakhlov, Yu.Simonov and B.S.Zou for useful discussions and clarifying remarks. We are grateful to Dr. Shi-Lin for drawing our attention to paper [7]. B.K. acknowledge the support from the grant Ssc-1774-2003.

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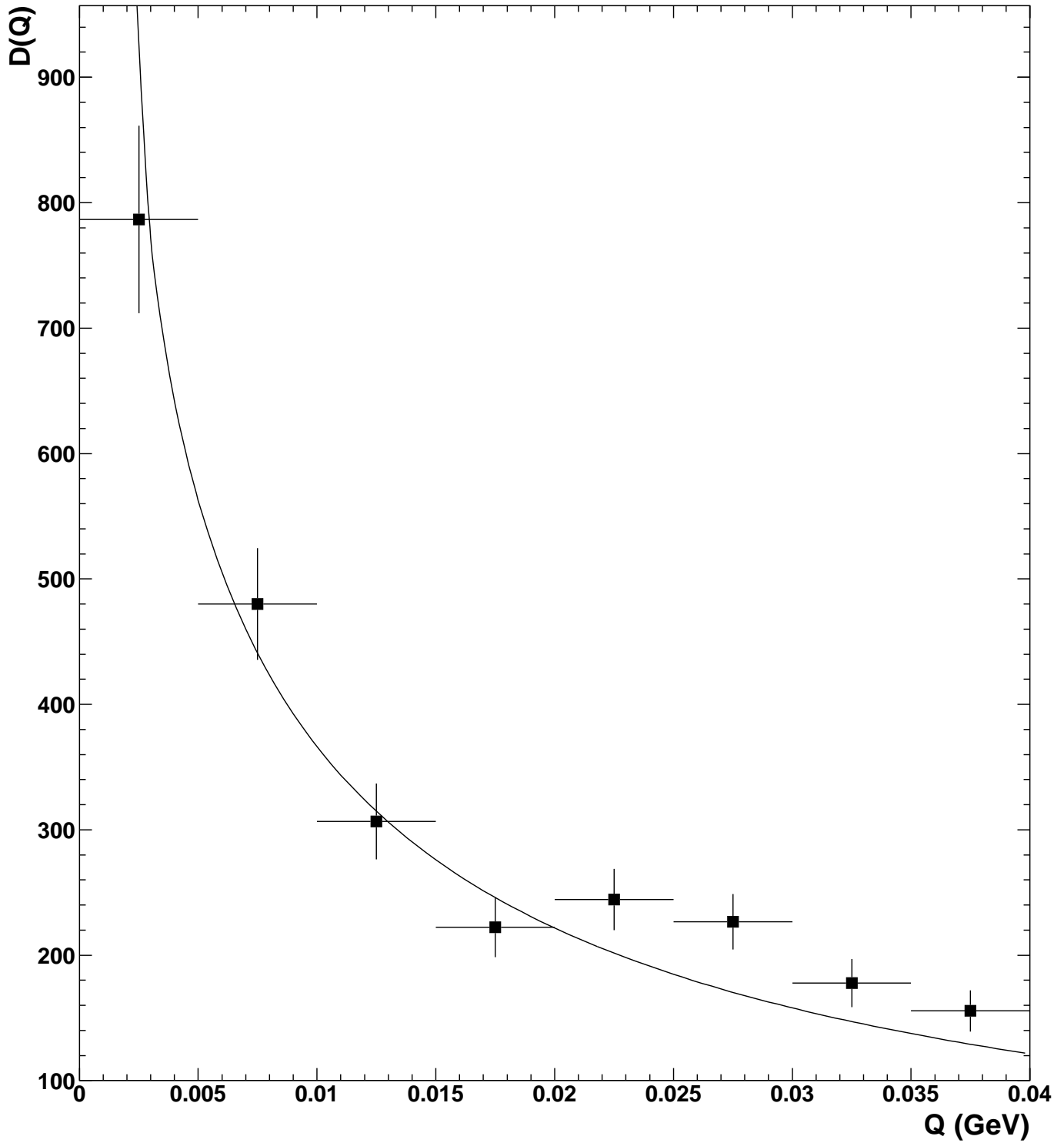


Fig. 1 The near-threshold behaviour of the enhancement function  $D(Q)$ ,  $Q = m_{23} - 2m$ . Solid curve corresponds to the parameter set (11). Experimental points are from Ref.[3].

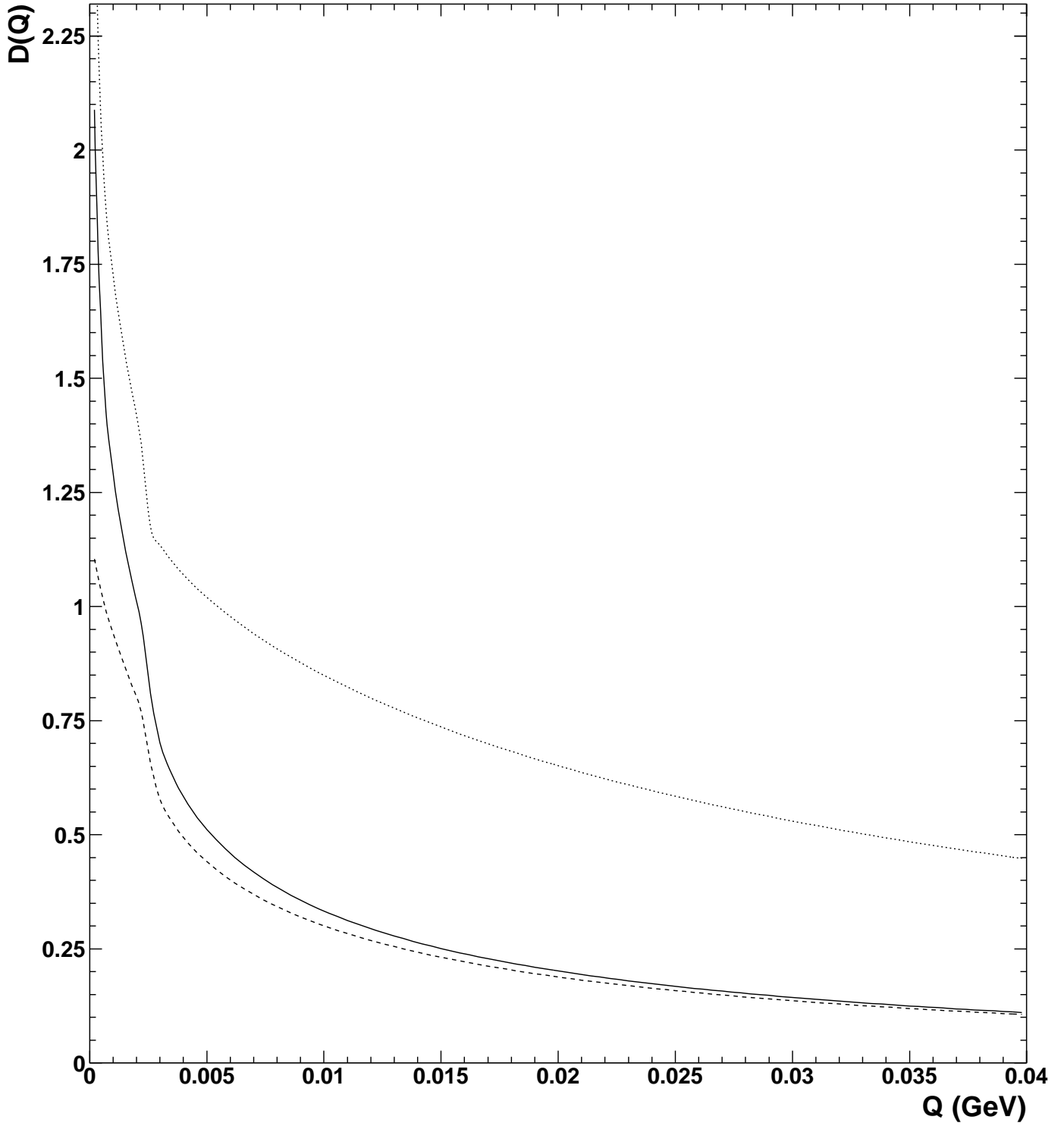


Fig. 2 The enhancement factor according to (6) with parameter set (11) – the solid curve, the same without Coulomb interaction – the dashed curve, without annihilation – the dotted curve.